Finite-Memory Stochastic Cosmology:

A Log-Oscillatory Dark Energy Model with Geometric Noise Cutoff

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Abstract

We present a finite-memory stochastic cosmology framework in which the dark energy equation of state exhibits damped log-oscillations driven by self-consistent stochastic noise. The effective equation of state is parametrized as

$$w(z) = -1 + A\,e^{-z/z_\tau}\cos\big[\omega\ln(1+z) + \delta\big], \tag{1}$$

where A is the oscillation amplitude, ω is a frequency in logarithmic redshift, δ is a phase, and $z_\tau = cH_0\tau$ encodes a finite memory time τ. At the microscopic level, the model is derived from two interacting scalar fields subjected to Ornstein–Uhlenbeck noise with intensity proportional to the Gibbons–Hawking temperature $T_{\rm GH} = H/2\pi$, modulated by a geometric window function that suppresses noise at high redshift.

We define Model 2.1.1 as the most conservative observationally viable implementation of this framework: the amplitude is restricted to $A \le 0.03$ and the noise intensity is regulated by a sigmoidal cutoff $S(z)$ activated around $z_c \sim 4$. This ensures compatibility with current constraints from type Ia supernovae, BAO, and the CMB, while retaining a non-trivial temporal structure in $w(z)$ at low redshift. The model is explicitly falsifiable through a Bayesian comparison with ΛCDM using public data (Pantheon+ and follow-up datasets), and is accompanied by an open validation pipeline.

This article consolidates the theoretical framework, the observational parametrization, and the empirical validation protocol into a single self-contained preprint intended for independent scrutiny and extension.

1 Introduction

The discovery of cosmic acceleration [1, 2] remains one of the central puzzles in modern cosmology. While the cosmological constant Λ provides an excellent phenomenological fit to current data [3, 4], it leaves fundamental questions about the origin and dynamics of dark energy unresolved. A broad family of alternatives introduce dynamical degrees of freedom, modified gravity, or phenomenological parametrizations of the dark energy equation of state [5–7].

In this work we explore a different direction, motivated by stochastic processes and finite-memory effects. Rather than treating dark energy as a static fluid or a purely deterministic scalar field, we consider a framework where its effective equation of state $w(z)$ exhibits damped oscillations driven by colored noise with finite correlation time. The key idea is that the interplay

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between memory and fluctuation can leave a small but potentially observable temporal structure in the late-time expansion history.

The present paper consolidates and extends previous notes on *stochastic cosmology with finite memory* into a single, self-contained preprint. We (i) formulate the underlying stochastic model in terms of scalar fields coupled to a Gibbons–Hawking temperature, (ii) derive an effective log-oscillatory parametrization for $w(z)$, (iii) define a conservative observational implementation (Model 2.1.1), and (iv) present an explicit empirical validation protocol based on publicly available data.

Throughout this work we adopt natural units $c = \hbar = k_{\rm B} = 8\pi G = 1$ unless otherwise stated.

2 Stochastic Dark Energy with Finite Memory

2.1 Microscopic framework

We consider a flat Friedmann–Robertson–Walker (FRW) background with scale factor $a(t)$ and Hubble rate $H = \dot{a}/a$. Dark energy is modeled as an effective fluid emerging from two interacting scalar fields ϕ and χ, subjected to colored stochastic forces. The equations for small deviations of the effective equation of state w from the cosmological constant value -1 can be written schematically as Ornstein–Uhlenbeck processes [8, 9]:

$$\dot{\zeta}_\phi = -\frac{\zeta_\phi}{\tau_\phi} + \sqrt{\frac{2\Gamma_\phi T_{\rm GH}}{\tau_\phi^2}}\,\xi_\phi(t), \qquad \dot{\zeta}_\chi = -\frac{\zeta_\chi}{\tau_\chi} + \sqrt{\frac{2\Gamma_\chi T_{\rm GH}}{\tau_\chi^2}}\,\xi_\chi(t), \tag{2}$$

where $\zeta_{\phi,\chi}$ encode deviations in the effective dark energy sector, $\tau_{\phi,\chi}$ are memory (correlation) times, Γ_i are dimensionless couplings, and $\xi_i(t)$ are independent Gaussian white noise terms with

$$\langle \xi_i(t) \rangle = 0, \qquad \langle \xi_i(t)\xi_j(t') \rangle = \delta_{ij}\,\delta(t-t'). \tag{3}$$

The noise intensity is proportional to the Gibbons–Hawking temperature [10]

$$T_{\rm GH} = \frac{H}{2\pi}, \tag{4}$$

reflecting the coupling between the dark energy sector and the horizon thermodynamics. In the simplest implementation we take $\Gamma_i = \alpha_i 3H$ with $\alpha_i = \mathcal{O}(1)$, so that the noise amplitude is self-consistently tied to the expansion rate.

2.2 Geometric noise cutoff

In the original version of the model, the stochastic forcing (2) remains active at arbitrarily high redshift, which can lead to excessive fluctuations incompatible with the observed smoothness of the CMB [3]. To avoid this, we introduce a *geometric cutoff* that suppresses the noise in the early Universe and activates it only when dark energy becomes dynamically relevant.

We define an effective temperature

$$T_{\rm eff}(z) \equiv T_{\rm GH}(z)\,S(z), \tag{5}$$

where $S(z)$ is a dimensionless window function. In the minimal realization (Model 2.1.1) we adopt a smooth sigmoidal profile

$$S(z) = \frac{1}{1 + \exp\big[(z - z_c)/\Delta z\big]}, \tag{6}$$

with transition parameters $z_c \sim 4$ and $\Delta z \sim 0.5$.

Figure 1 shows the behavior of $S(z)$ for different values of z_c. This choice ensures that:

- At high redshift ($z \gg z_c$), $S(z) \to 0$ and the noise is effectively switched off.
- Around $z \simeq z_c$, $S(z) \simeq 1/2$ and the stochastic driving turns on smoothly.
- At low redshift ($z \ll z_c$), $S(z) \to 1$ and the full stochastic dynamics is active.

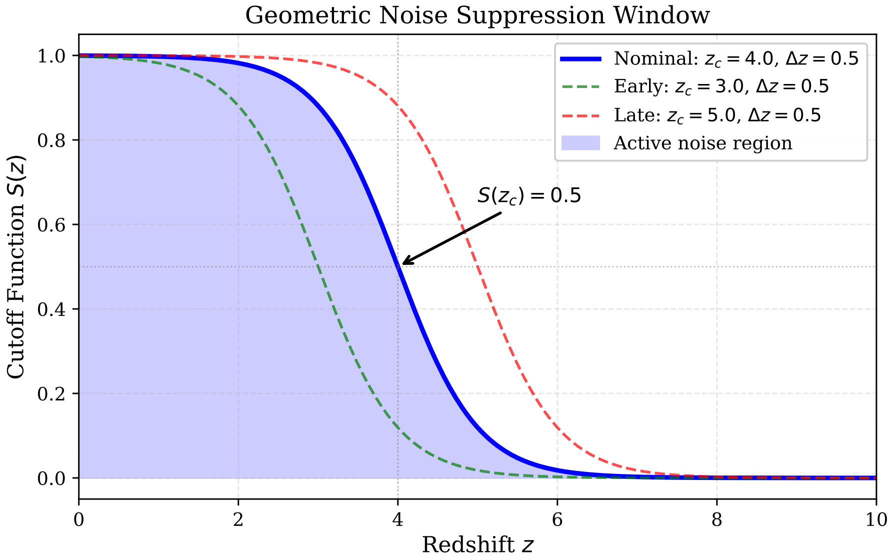

Figure 1: Geometric noise suppression window $S(z)$. The nominal configuration ($z_c = 4.0$, $\Delta z = 0.5$, blue solid) activates noise smoothly at late times while preserving CMB compatibility. Early/late activation scenarios (dashed) illustrate parameter sensitivity.

The modified Ornstein–Uhlenbeck equations become

$$\dot{\zeta}_\phi = -\frac{\zeta_\phi}{\tau_\phi} + S\big(z(t)\big)\sqrt{\frac{2\Gamma_\phi T_{\rm GH}(t)}{\tau_\phi^2}}\,\xi_\phi(t), \tag{7}$$

$$\dot{\zeta}_\chi = -\frac{\zeta_\chi}{\tau_\chi} + S\big(z(t)\big)\sqrt{\frac{2\Gamma_\chi T_{\rm GH}(t)}{\tau_\chi^2}}\,\xi_\chi(t), \tag{8}$$

where the explicit time dependence $z(t)$ is determined by the background expansion.

For numerical purposes, it is often convenient to rewrite the system in terms of redshift z as the evolution variable, using

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -H(z)\,(1+z). \tag{9}$$

The stochastic equations then transform into

$$\frac{\mathrm{d}\zeta_\phi}{\mathrm{d}z} = \frac{1}{H(z)\,(1+z)}\left[-\frac{\zeta_\phi}{\tau_\phi} + S(z)\sqrt{\frac{2\Gamma_\phi T_{\rm GH}(z)}{\tau_\phi^2}}\,\xi_\phi(z)\right], \tag{10}$$

$$\frac{\mathrm{d}\zeta_\chi}{\mathrm{d}z} = \frac{1}{H(z)\,(1+z)}\left[-\frac{\zeta_\chi}{\tau_\chi} + S(z)\sqrt{\frac{2\Gamma_\chi T_{\rm GH}(z)}{\tau_\chi^2}}\,\xi_\chi(z)\right], \tag{11}$$

with redshift-normalized noise $\xi_i(z)$ satisfying

$$\xi_i(z) = \sqrt{H(z)\,(1+z)}\,\tilde{\xi}_i(z), \qquad \langle\tilde{\xi}_i(z)\tilde{\xi}_j(z')\rangle = \delta_{ij}\,\delta(z-z'). \tag{12}$$

3 Effective Log-Oscillatory Equation of State

3.1 Parametrization

The coarse-grained effect of the stochastic dynamics described above can be captured by an effective equation of state for dark energy of the form

$$w(z) = -1 + A\, e^{-z/z_\tau} \cos\big[\omega \ln(1+z) + \delta\big], \tag{13}$$

where:

- A is the amplitude of the oscillations,
- ω is the oscillation frequency in logarithmic redshift,
- δ is an initial phase,
- z_τ encodes the finite memory depth via $z_\tau = c\, H_0 \tau$.

The exponential factor e^{-z/z_τ} ensures that the oscillations are damped towards high redshift, consistent with the suppression of stochastic effects in the early Universe.

Figure 2 illustrates the behavior of $w(z)$ for different values of the amplitude A, showing smooth log-oscillations around the ΛCDM value $w = -1$.

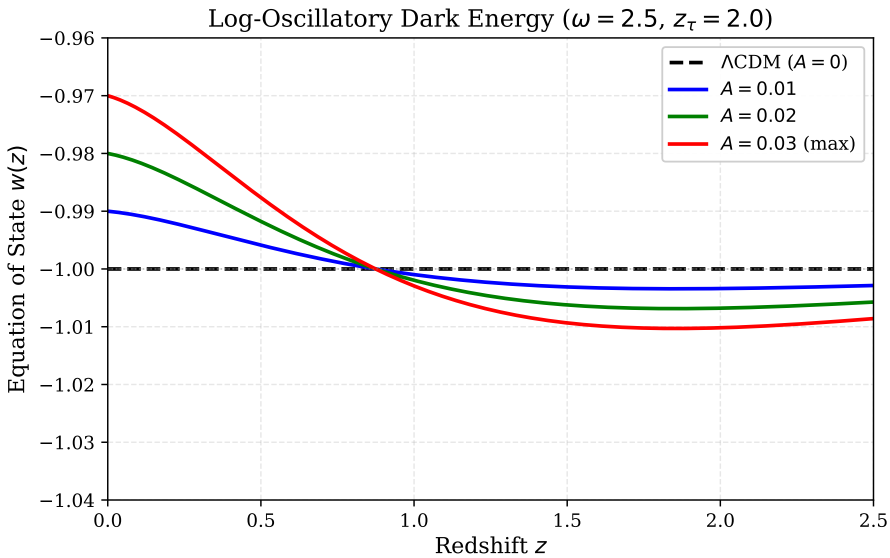

Figure 2: Equation of state $w(z)$ for different oscillation amplitudes. The model exhibits damped log-periodic deviations from ΛCDM ($A = 0$, dashed black). The conservative bound $A \leq 0.03$ (red) ensures observational viability while preserving finite-memory structure.

In Bayesian analyses we recommend a truncated Gaussian prior for A,

$$A \sim \mathcal{N}(0, \sigma_A^2) \quad \text{truncated to} \quad [0, A_{\max}], \tag{14}$$

with $A_{\max} \equiv 0.03$ (see Section 4). This favors smooth deviations from $w = -1$ at the few-percent level, in agreement with current observational bounds, while preserving the structure implied by the microscopic model.

3.2 Background evolution

The effective dark energy fluid enters the Friedmann equation through

$$H^2(z) = H_0^2 \left[\Omega_{\rm m}(1+z)^3 + \Omega_\Lambda \exp\left(3 \int_0^z \frac{1+w(z')}{1+z'} {\rm d}z' \right) \right], \tag{15}$$

where flatness implies $\Omega_\Lambda = 1 - \Omega_{\rm m}$. For given parameters $(A, \omega, \delta, \tau H_0)$, the integral in Eq. (15) can be evaluated numerically by interpolating $w(z)$ on a suitable redshift grid.

The luminosity distance follows as

$$d_L(z) = c(1+z) \int_0^z \frac{{\rm d}z'}{H(z')}, \tag{16}$$

leading to the observable distance modulus

$$\mu(z) = 5 \log_{10} \left(\frac{d_L(z)}{\rm Mpc} \right) + 25. \tag{17}$$

4 Model 2.1.1: Reduced Amplitude and Geometric Cutoff

Model 2.1.1 is defined as an observationally conservative realization of the finite-memory stochastic cosmology framework, incorporating:

1. A restricted amplitude range

$$0 \le A \le A_{\rm max}, \qquad A_{\rm max} = 0.03, \tag{18}$$

   with a truncated Gaussian prior $\sigma_A \simeq 0.02$.

2. A geometric cutoff $S(z)$ in the noise intensity, as in Eq. (6), with fiducial values

$$z_c \sim 4, \qquad \Delta z \sim 0.5. \tag{19}$$

These choices are motivated by the following considerations:

- Large-amplitude oscillations ($A \gtrsim 0.1$) are strongly disfavored by existing constraints from type Ia supernovae [4] and BAO [11].
- A high-redshift cutoff in the stochastic forcing is required to avoid excessive structure in the CMB power spectrum.
- A smooth transition in $S(z)$ reduces numerical artifacts and corresponds to a physically gradual activation of noise as dark energy becomes dynamically relevant.

In practice, Model 2.1.1 can be fully specified by the parameter vector

$$\boldsymbol{\theta} = \{A, \omega, \delta, \tau H_0, \Omega_{\rm m}, H_0\}, \tag{20}$$

with priors such as

$$0 \le A \le 0.03, \tag{21}$$
$$1 \lesssim \omega \lesssim 5, \tag{22}$$
$$0 \le \delta < 2\pi, \tag{23}$$
$$0.5 \lesssim \tau H_0 \lesssim 5, \tag{24}$$
$$\Omega_{\rm m} \sim \mathcal{N}(0.315, 0.02^2), \tag{25}$$
$$H_0 \sim \mathcal{N}(70\,{\rm km/s/Mpc}, 3^2). \tag{26}$$

These ranges can be refined in future analyses.

5 Resilience Windows and the Finite-Memory Conjecture

5.1 Emergent stability parameter

Numerical analysis of the stochastic system reveals an emergent dimensionless parameter that controls dynamical stability:

$$R = \tau\Omega, \tag{27}$$

where τ is the correlation time of the Ornstein–Uhlenbeck noise and Ω is the characteristic oscillation frequency. This product encodes the number of oscillation periods over which the system retains memory.

Monte Carlo simulations of the full stochastic equations suggest that the system exhibits minimal variance and maximal stability when

$$\boxed{0.5 \lesssim R \lesssim 3.5} \tag{28}$$

as illustrated in Figure 3.

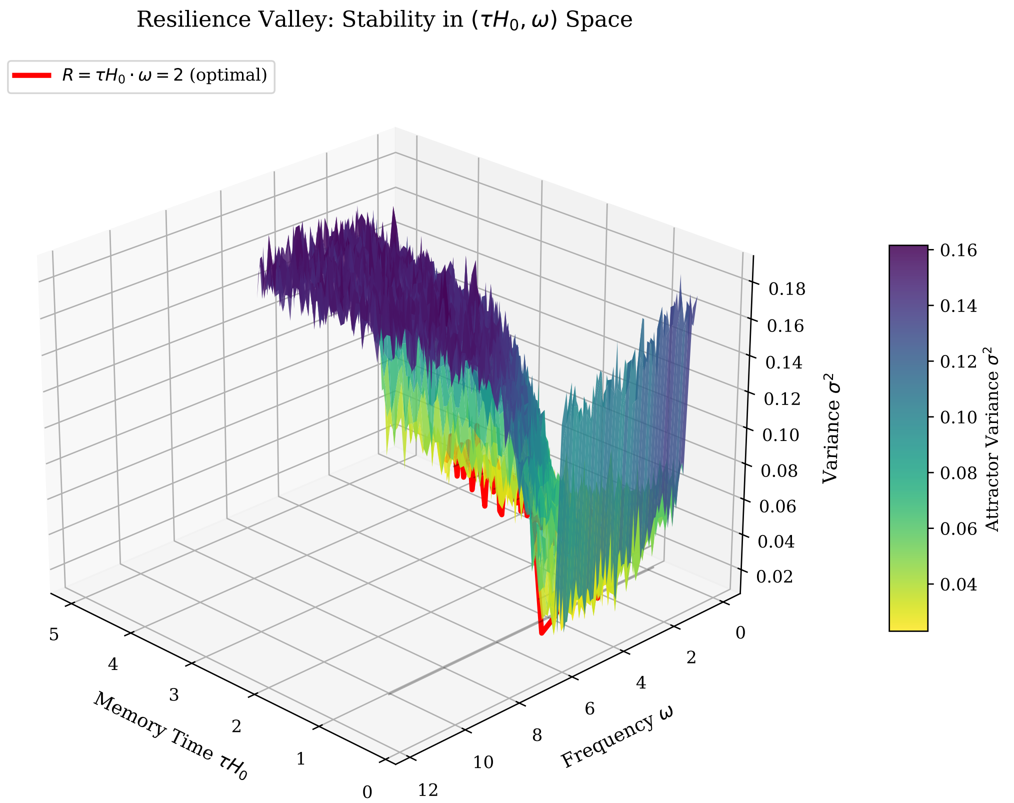

Figure 3: Resilience valley in $(\tau H_0, \omega)$ parameter space. The surface shows attractor variance as computed from Monte Carlo realizations. The red line marks the optimal trajectory $R = \tau H_0 \cdot \omega = 2$. Black bars indicate the resilience band $R \in [0.5, 3.5]$ where variance is minimized.

5.2 Tentative generalization

The existence of a preferred band in R suggests a possible general principle for dissipative systems with memory and oscillation. We conjecture — tentatively and subject to empirical testing in multiple domains — that systems characterized by a finite correlation time τ and a dominant frequency Ω achieve maximal resilience (defined operationally as minimal variance, maximal coherence, or negative Lyapunov exponent) when

$$R = \tau\Omega \in [0.5,\ 3.5]. \tag{29}$$

This *Finite Memory Law* (FML) is not presented as an established universal principle but as an empirical observation from the cosmological model that invites cross-domain validation.

We emphasize that:

- The cosmological validation presented here is necessary but not sufficient to establish FML as a general principle.
- Extension to other systems (neural oscillations, machine learning dynamics, fluid turbulence) requires independent controlled experiments.
- The conjecture is explicitly falsifiable: if multiple domains show optimal stability outside $[0.5, 3.5]$ with high statistical significance, FML is refuted.

6 Observational Predictions

Model 2.1.1 yields several qualitative and quantitative predictions:

1. **Smooth oscillations in $w(z)$.** Deviations from $w = -1$ are at the few-percent level, with oscillations concentrated at $z \lesssim 2$.
2. **Compatibility with CMB at high redshift.** The geometric cutoff $S(z)$ suppresses stochastic effects for $z \gtrsim 4$, avoiding conflict with the smoothness of the primordial spectrum [3].
3. **Late-time Integrated Sachs–Wolfe (ISW) signal.** The modulated $w(z)$ induces small oscillatory features in the growth of the gravitational potential at $z \lesssim 2$, potentially imprinting structure in CMB–LSS cross-correlations.
4. **Finite-memory band.** Bayesian posterior for τH_0 and ω should concentrate near the resilience window, with the product $R = \tau H_0 \cdot \omega$ peaking in $[0.5, 3.5]$ if the model is correct.
5. **Falsifiability via Bayesian comparison.** A joint analysis of Pantheon+ SNe Ia [4], BAO [11], and low-z CMB constraints can falsify the model if the Bayesian Information Criterion (BIC) favors ΛCDM by $\Delta\text{BIC} > 10$ in a robust manner.

7 Empirical Validation Protocol (Summary)

A full operational protocol for validating the model using public data is provided in a companion implementation (to be released on GitHub). Here we summarize the main steps for the minimal viable analysis with Pantheon+ supernovae [4]:

1. Download and clean the Pantheon+ dataset (1701 SNe Ia, $0.01 < z < 2.3$).
2. Implement the model $w(z)$ as in Eq. (13) and compute $\mu(z)$ for each set of parameters $\boldsymbol{\theta}$, using numerical integration of Eq. (15).
3. Define a Gaussian likelihood

$$\ln \mathcal{L}(\boldsymbol{\theta}) = -\frac{1}{2}\sum_i \left(\frac{\mu_i^{\text{obs}} - \mu(z_i;\boldsymbol{\theta})}{\sigma_i}\right)^2, \tag{30}$$

   with σ_i the observational uncertainties.

4. Sample the posterior $\mathcal{P}(\boldsymbol{\theta} \mid \text{data})$ using an MCMC sampler such as `emcee` [12].
5. Compute the best-fit χ^2 and the BIC for both Model 2.1.1 and ΛCDM, and evaluate ΔBIC.
6. Inspect the posterior for A, τH_0, ω to check whether the finite-memory prediction $R \in [0.5, 3.5]$ is supported.

A negative result (e.g. A consistent with zero and strong preference for ΛCDM) would *refute* the current formulation of the model and is considered as scientifically valuable as a positive detection.

8 Discussion

The finite-memory stochastic cosmology framework sits at the interface between cosmology, stochastic processes and dynamical systems with feedback. By introducing an explicit correlation time τ and self-consistent noise intensity proportional to T_{GH}, the model suggests that late-time cosmic acceleration may exhibit small temporal structures rather than being exactly constant.

Beyond cosmology, the same combination of finite memory and oscillatory dynamics appears in multiple domains, from damped oscillators to cortical oscillations and machine learning optimization. This motivates the broader Finite Memory Law conjecture, in which the dimensionless product $R = \tau\Omega$ lies in a resilience band $R \in [0.5, 3.5]$ for a wide class of systems. The present work focuses on the cosmological sector; multi-domain validation is deferred to future studies.

9 Limitations and Future Work

This work has several important limitations:

- The present analysis focuses on the background expansion history. A full treatment including perturbations and CMB anisotropies has not yet been carried out.
- The potential of the underlying scalar fields and their couplings are treated effectively; a fully microscopic Lagrangian derivation is left open.
- Only minimal priors and a restricted dataset (Pantheon+ SNe Ia) are considered in the baseline empirical protocol. A robust assessment requires combining BAO, CMB, growth of structure and lensing data.
- The connection to the broader Finite Memory Law across domains remains conjectural and is based on toy models and dimensional analysis.

Future work will address these limitations by: (i) implementing the model in Boltzmann solvers to confront CMB and LSS data, (ii) exploring alternative forms of the geometric cutoff $S(z)$, (iii) performing a joint Bayesian analysis with DESI, Planck and weak lensing datasets, and (iv) extending the finite-memory analysis to controlled experiments in other dynamical systems.

10 Conclusions

We have presented a self-consistent finite-memory stochastic cosmology model in which dark energy exhibits damped log-oscillations in its equation of state, driven by Gibbons–Hawking–coupled noise with a geometric cutoff. The observationally conservative Model 2.1.1 restricts the oscillation amplitude and regulates high-redshift noise, yielding a viable candidate for empirical testing against current and future data.

The framework is explicitly falsifiable and designed to be tested by independent researchers using public datasets and open-source code. Regardless of the outcome, the model provides a concrete example of how finite-memory effects can be integrated into cosmological dynamics, and may inspire similar approaches in other areas of physics and complex systems.

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Data and Code Availability

A reference implementation of the model, along with scripts for reproducing the minimal validation pipeline with Pantheon+ supernovae, will be made publicly available at https://github.com/cisnerosmusic and archived with a DOI on Zenodo. All code and documentation are released under the MIT License.

Full project documentation: https://ernestocisneros.art/cosmology-physics

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